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Upper and lower bounds for the blow up time for sixth-order parabolic-type equations with time dependent coefficient

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Abstract

In this work, we consider the sixth-order Parabolic-type equations with time dependent coefficient. We proved a lower and an upper bound for the blow-up time is determined by means of a differential inequality argument when blow up occurs.

1 Introduction

This work, we study the following sixth-order Parabolic-type equations with time dependent coefficient with initial-boundary value:

$$\begin{cases} z_t + z_{xxxx} - z_{xxxxxx} + z_{xxxxt} = \alpha(t)\, g(z_x)_x, & x \in \Omega,\ t > 0, \\ z(x,0) = z_0(x),\ \ z_t(x,0) = z_1(x), & x \in \partial\Omega,\ t > 0, \\ z(0,t) = z(1,t) = z_{xx}(0,t) = z_{xx}(1,t) = z_{xxxx}(0,t) = z_{xxxx}(1,t) = 0, & x \in \Omega,\ t \geq 0, \end{cases} \quad (1.1)$$

here $\Omega \subset R^n\ (n \geq 1)$ is a domain with smooth boundary $\partial\Omega$ in R^n. The coefficient $\alpha(t)$ is assumed a strictly continuously and positive differentiable function in $\Omega = (0,1)$. The nonlinear smooth function $g(s)$ satisfies the following hypotheses:

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(H1) $g(s) = 0$, $g(s)$ is monotone and is convex for $s > 0$, concave for $s < 0$,
(H2) $|g(s)| \leq \beta |s|^q$, $\beta > 0$, $1 < q < +\infty$, $\forall s \in R$,
(H3) $|q+1| G(s) \leq sg(s)$, for some $q > 1$, $\forall s \in R$, $G(s) = \int_0^s f(\tau) d\tau$.

This mathematical model has emerged in the creation of spatially periodic patterns in bistable systems as well as a framework for understanding phase front behavior in materials that are transitioning among liquid and solid states [3].

Anbu et al. [3] studied the sixth-order partial differential equation (PDE)

$$z_t - z_{xxxxxx} + Az_{xxxx} - Bz_{xx} = f(z).$$

They demonstrate the presence of global solutions for the given equation by employing Dirichlet-Neumann type boundary conditions. Additionally, they deduce an upper limit for the blow-up time of the solution. Finally, they acquire a lower limit for the blow-up time of the solution through the application of the first-order differential inequality technique in the event of blow-up.

Gyulov et al. [6] concerned with the following problem:

$$z^{(6)} + Az^{(4)} + Bz'' + Cz = f(t, z).$$

They proved existence for a semilinear sixth-order ordinary differential equation (ODE). Later, Zhang and An [17] studied the existence and multiplicity of positive solutions of the same equation. Also, Li et al. [9] established the existence of positive solutions of the same equation.

Tersian and Chaparova [15] studied the existence of periodic solutions of the sixth-order ODE

$$z^{(6)} + Az^{(4)} + Bz'' + z - z^3 = 0.$$

Han [7] concerned the blow-up property of solutions to the following fourth-order parabolic equation with a general nonlinearity

$$z_t + \Delta^2 z = k(t) f(z).$$

He showed, under certain conditions on the initial data, that the solutions to this problem blow up in finite time, using differential inequalities. Also, upper and lower bounds for the blow-up time are derived when blow-up occurs.

Di and Shang [4] considered the metaparabolic equation with time dependent coefficients

$$z_t - z_{xx} - z_{xxt} + z_{xxxx} = k(t) f(z_x)_x. \tag{1.2}$$

They proved an upper and lower bound for blow-up time. If differentiating (1.2) with respect to x and integrating by parts and take $k(t) = 1$, then we have

$$z_{xt} - z_{xxxt} + z_{xxxxx} = \varphi(z_x)_{xx}.$$

It is the well-known viscous Chan- Hilliard equation, here $\varphi(z_x) = f(z_x) + z_x$.

Pişkin and Fidan [12] considered the variable coefficients wave equation

$$z_{tt} - \Delta z - \Delta z_t + \mu_1(t) |z_t|^{p-2} z_t = \mu_2(t) |z|^{q-2} z.$$

They proved the blow up of solutions.

Wu [16] considered the Petrovsky equation with variable coefficients

$$z_{tt} + \Delta^2 z - \Delta z - \omega \Delta z_t + \alpha(t) z_t = |z|^{p-2} z,$$

and obtained the blow-up result with lower and upper bounded.

Pişkin and Fidan [13] concerned with the following problem:

$$z_{tt} - div\left(|\nabla z|^{m-2} \nabla z\right) + \Delta^2 z + \mu_1(t) |z_t|^{p-2} z_t = \mu_2(t) |z|^{q-2} z.$$

They prove the blow up of solutions for finite time with negative initial energy.

In our research, we employed various types of Dirichlet-Neumann boundary conditions in conjunction with a general nonlinear term. Additionally, we derived the primary outcomes of this paper using a methodology distinct from those discussed in prior works. While some of the literature has addressed blow-up solutions for higher-order PDE's and coupled parabolic systems, to the best of our knowledge, there is currently no article available that specifically explores the finite-time blow-up solutions for a sixth-order PDE with a general nonlinearity term $f(z)$. Consequently, we endeavored to investigate and present new and noteworthy findings in this regard. For a more in-depth exploration of blow-up phenomena in higher-order PDEs, readers are encouraged to consult the book by Galaktionov [5].

Blow up phenomena commonly arise in solutions to reaction-diffusion partial differential equations of various types. A recent comprehensive overview of these methods can be found in the monograph by Al'shin et al. [2], Hu [8] and Pişkin [11].

Motivated by above-mentioned papers, in this paper, we investigate to prove the upper and lower bounds for the blow up time of solutions for problem (1.1), which was not previously studied, where we study sixth-order parabolic equation with time dependent coefficient source terms $\alpha(t) g(z_x)_x$. Our study is motivated by Di and Shang [4].

The rest of the work is as follows: In Part 2, we give some assumptions needed in this work and under suitable conditions, we obtain an upper bounds for the blow up time. In Part 3, under suitable conditions, we obtain a lower bounds for the blow up time.

2 Preliminaries

In this part, we present certain lemmas and assumptions required for the formulation and proof of our results. Let $\|.\|, \|.\|_p$ and $\|.\|_{W^{m,p}(\Omega)}$ indicate the typical $L^2(\Omega)$, $L^p(\Omega)$ and $W^{m,p}(\Omega)$ norms (see [1, 14]).

$$\mathcal{W} = \left\{ z \in H^4(\Omega) : z(0,t) = z(1,t) = z_{xx}(0,t) = z_{xx}(1,t) = z_{xxxx}(0,t) \right.$$
$$\left. = z_{xxxx}(1,t) = 0 \right\}.$$

We define the following functionals

$$J(t) = \frac{1}{2}\|z_{xx}\|^2 + \frac{1}{2}\|z_{xxx}\|^2,$$

and

$$I(t) = \|z_{xx}\|^2 + \|z_{xxx}\|^2.$$

The functional E of the problem (1.1) is as follows:

$$E(t) = \frac{1}{2}\|z_{xx}\|^2 + \frac{1}{2}\|z_{xxx}\|^2 + \int_0^1 k(t) G(z_x)\, dx. \tag{2.1}$$

Lemma 2.1. *Assume that z is a solution to the problem (1.1). Then the energy of problem (1.1) defined by (2.1) satisfies and*

$$E'(t) = -\|z_t\|^2 - \|z_{xxt}\|^2 \leq 0. \tag{2.2}$$

Proof. Multiplying the eq. (1.1) by z_t integrate it over Ω, apply Green's formula, we obtain

$$E(t) - E(0) = -\int_0^t \|z_t\|^2 d\tau - \int_0^t \|z_{xxt}\|^2 d\tau, \text{ for } t \geq 0. \tag{2.3}$$

□

Then we define the following auxiliary functions:

$$\mathcal{K}(t) = \|z\|^2 + \|z_{xx}\|^2, \tag{2.4}$$

and

$$\mathcal{M}(t) = \frac{1}{2}\|z_{xx}\|^2 + \frac{1}{2}\|z_{xxx}\|^2 + \int_0^1 \alpha(t) G(z_x)\, dx. \tag{2.5}$$

3 Upper bound for blow-up time

In this part, we show an upper bound for blow-up time $\mathcal{T}^*$ of the solution for problem (1.1).

Theorem 3.1. *Let $z_0 \in W^{2,q+1}(\Omega) \cap \mathcal{W}$ and g satisfy the hypotheses (H1)-(H3). Let's assume that the data for the problem (1.1) satisfies the following conditions:*

$$g(t) < 0,\ g'(t) \leq 0, \tag{3.1}$$

$$\mathcal{M}(0) = \frac{1}{2}\|z_{0xx}\|^2 + \frac{1}{2}\|z_{0xxx}\|^2 + \int_0^1 \alpha(0) G(z_{0x})\, dx < 0. \tag{3.2}$$

Then, we infer that the solutions z of problem (1.1) can't exist for all time. Therefore, blow-up time $\mathcal{T}^*$ of problem (1.1) is given by

$$\mathcal{T}^* \leq \frac{\mathcal{K}(0)}{(1-q^2)\mathcal{M}(0)}, \tag{3.3}$$

here $\mathcal{K}(0) = \|z_0\|^2 - \|z_{0xx}\|^2$

Proof. Firstly, by taking the first order derivative of (2.5), we get

$$\begin{aligned} \mathcal{M}'(t) &= \int_0^1 \alpha'(t) G(z_x)\, dx + \int_0^1 \alpha(t) g(z_x) z_{xt} dx + \int_0^1 z_{xx} z_{xxt} dx + \int_0^1 z_{xxx} z_{xxxt} dx, \\ &= \int_0^1 \alpha'(t) G(z_x)\, dx + \int_0^1 \left[-z_{xxxxxx} + z_{xxxx} - \alpha(t) g(z_x)_x\right] z_t dx, \end{aligned} \tag{3.4}$$

where $G(s)$ is nonnegative for all $s \in R$ under the hypothesis (H1) (the Lemma

2.2 of [10]). Therefore, combining (1.1), (3.1) and (3.4), we get

$$\begin{aligned} \mathcal{M}'(t) &= \int_0^1 \alpha'(t)\, G(z_x)\, dx + \int_0^1 \left[-z_{xxxxxx} + z_{xxxx} - \alpha(t)\, g(z_x)_x\right] z_t dx, \\ &= -\|z_t\|^2 - \|z_{xxt}\|^2 < 0. \end{aligned} \tag{3.5}$$

It then follows that $\mathcal{M}(t)$ function (2.5) which is a non-increasing function, we obtain

$$\mathcal{M}(t) \leq \mathcal{M}(0) < 0. \tag{3.6}$$

For the sake of simplicity, let's define the function $F(t) = -\mathcal{M}(t)$ for all t in the interval $[0, \infty)$. Referring to equations (3.5) and (3.6), we observe:

$$F'(t) = \mathcal{M}'(t) \geq \|z_t\|^2 + \|z_{xxt}\|^2 > 0, \tag{3.7}$$

and

$$F(t) \geq F(0) > 0. \tag{3.8}$$

Differentiating $\mathcal{K}(t)$ with respect to t, based on the definition of $F(t)$ and hypothesis (H1), we obtain

$$\begin{aligned} \mathcal{K}'(t) &= 2\int_0^1 z z_t dx + 2\int_0^1 z_{xx} z_{xxt} dx \\ &= 2\int_0^1 z\left[-z_{xxxx} - z_{xxxxt} + z_{xxxxxx} + \alpha(t)\, g(z_x)_x\right] dx + 2\int_0^1 z_{xx} z_{xxt} dx \\ &= -2\|z_{xx}\|^2 - 2\|z_{xxx}\|^2 - 2\int_0^1 \alpha(t)\, g(z_x)\, z_x dx \\ &\geq -2\|z_{xx}\|^2 - 2\|z_{xxx}\|^2 - 2(q+1)\int_0^1 \alpha(t)\, G(z_x)\, dx \\ &= 2(q+1)\left[-\frac{1}{q+1}\|z_{xx}\|^2 - \frac{1}{q+1}\|z_{xxx}\|^2 - \int_0^1 \alpha(t)\, G(z_x)\, dx\right] \\ &\geq 2(q+1)\left[-\frac{1}{2}\|z_{xx}\|^2 - \frac{1}{2}\|z_{xxx}\|^2 - \int_0^1 \alpha(t)\, G(z_x)\, dx\right] \\ &= 2(q+1)\, F(t). \end{aligned} \tag{3.9}$$

If we multiply $K(t)$ by $F'(t)$, we obtain

$$\begin{aligned}\mathcal{K}F' &\geq \int_0^1 \left[z^2 + z_{xx}^2\right] dx \int_0^1 \left[z_t^2 + z_{xxt}^2\right] dx \\ &= \int_0^1 z^2 dx \int_0^1 z_t^2 dx + \int_0^1 z^2 dx \int_0^1 z_{xxt}^2 dx \\ &\quad + \int_0^1 z_{xx}^2 dx \int_0^1 z_t^2 dx + \int_0^1 z_{xx}^2 dx \int_0^1 z_{xxt}^2 dx. \qquad (3.10)\end{aligned}$$

Utilizing the Schwarz's and Young's inequalities, we obtain

$$\int_0^1 z z_t dx \leq \left(\int_0^1 z^2 dx\right)^{\frac{1}{2}} \left(\int_0^1 z_t^2 dx\right)^{\frac{1}{2}}, \qquad (3.11)$$

$$\int_0^1 z_{xx} z_{xxt} dx \leq \left(\int_0^1 z_{xx} dx\right)^{\frac{1}{2}} \left(\int_0^1 z_{xxt} dx\right)^{\frac{1}{2}}, \qquad (3.12)$$

and

$$\int_0^1 z z_t dx \int_0^1 z_{xx} z_{xxt} dx \leq \frac{1}{2} \int_0^1 z^2 dx \int_0^1 z_{xxt}^2 dx + \frac{1}{2} \int_0^1 z_{xx}^2 dx \int_0^1 z_t^2 dx. \qquad (3.13)$$

By substituting (3.11)-(3.13) into (3.10), we can deduce from (3.9) that

$$\begin{aligned}\mathcal{K}F' &\geq \int_0^1 \left[z^2 + z_{xx}^2\right] dx \int_0^1 \left[z_t^2 + z_{xxt}^2\right] dx \\ &\geq \frac{1}{4} \left(\mathcal{K}'(t)\right)^2 \\ &\geq \frac{q+1}{2} \mathcal{K}' F. \qquad (3.14)\end{aligned}$$

The inequality above can be expressed as follows

$$\left(F \mathcal{K}^{-\frac{q+1}{2}}\right)' = \mathcal{K}^{-\frac{q+3}{2}} \left\{\mathcal{K}F' - \frac{q+1}{2} \mathcal{K}' F\right\} \geq 0. \qquad (3.15)$$

Integrating (3.15) with respect to t, we have

$$F(t) \left(\mathcal{K}(t)\right)^{-\frac{q+1}{2}} \geq F(0) \left(\mathcal{K}(0)\right)^{-\frac{q+1}{2}} = \mathcal{N}. \qquad (3.16)$$

From (3.9) and (3.16), it follows that

$$\frac{1}{1-q^2}\left(\mathcal{K}^{-\frac{q+1}{2}}\right)' = \frac{1}{2(q+1)}\mathcal{K}'\mathcal{K}^{-\frac{q+1}{2}} \geq F\mathcal{K}^{-\frac{q+1}{2}} \geq \mathcal{N}. \tag{3.17}$$

Integrating (3.17), with respect to t, we get

$$(\mathcal{K}(t))^{\frac{1-q}{2}} \leq (\mathcal{K}(0))^{\frac{1-q}{2}} - \left(q^2-1\right)\mathcal{N}t, \tag{3.18}$$

according to (3.18), we obtain

$$\mathcal{K}(t) \geq \frac{1}{\left[(\mathcal{K}(0))^{\frac{1-q}{2}} - (q^2-1)\mathcal{N}t\right]^{\frac{2}{q-1}}}. \tag{3.19}$$

Clearly, the inequality stated above cannot be satisfied for all $t > 0$. In fact, equation (3.19) results in the upper bound $\frac{\mathcal{K}(0)}{(1-q^2)\mathcal{M}(0)}$ for $\mathcal{T}^*$. □

4 Lower bound for blow-up time

In this part, we show a lower bound for blow-up time $\mathcal{T}^*$ of the solution for problem (1.1).

Theorem 4.1. *Assume that z be a blow-up solution to problem (1.1) and let g satisfy the hypotheses (H1)-(H3). Additionally, assume that $z_0 \in W^{2,q+1}(\Omega) \cap \mathcal{W}$ and $\alpha(t) < 0$ satisfies the condition*

$$\frac{\alpha'(t)}{\alpha(t)} \leq \eta, \text{ for all } t \geq 0 \tag{4.1}$$

for some constant $\eta \geq 0$. Under these conditions, we can conclude that the auxiliary function

$$\mathcal{N}(t) = (-\alpha(t))^{\frac{2}{q-1}}\left[\|z_{xx}\|^2 + \|z_{xxxx}\|^2\right], \tag{4.2}$$

at a finite time T^*, t becomes unbounded.

Furthermore, a lower bound for the blow-up time T^* can be estimated by

$$T^* \geq \begin{cases} \frac{2\mathcal{N}(0)^{1-q}}{(q-1)\beta^2\gamma^{2q}}, & \text{if } \eta = 0, \\ \frac{1}{2\eta}\ln\left(\frac{4\eta\mathcal{N}(0)^{1-q}}{(q-1)\beta^2\gamma^{2q}}\right), & \text{if } \eta > 0, \end{cases} \tag{4.3}$$

here C_* is the optimal constant that satisfies the inequality $\|z_{xx}\|_{2q} \leq C_* \|z_{xxxx}\|$.

Proof. By taking the first order derivative of (4.2), we get

$$\begin{aligned}
\mathcal{N}'(t) &= \frac{2}{q-1}(-\alpha(t))^{\frac{3-q}{q-1}}\left(-\alpha'(t)\right)\left[\int_0^1 z_{xx}^2 dx + \int_0^1 z_{xxxx}^2 dx\right] \\
&\quad + (-\alpha(t))^{\frac{2}{q-1}}\left[\int_0^1 z_{xx} z_{xxt} dx + \int_0^1 z_{xxxx} z_{xxxxt} dx\right] \\
&\leq \frac{2\eta}{q-1}\mathcal{N}(t) + 2(-\alpha(t))^{\frac{2}{q-1}} \int_0^1 [z_t + z_{xxxxt}] z_{xxxx} dx \\
&= \frac{2\eta}{q-1}\mathcal{N}(t) + 2(-\alpha(t))^{\frac{2}{q-1}} \int_0^1 [z_{xxxx} + z_{xxxxxx} + \alpha(t) g(z_x)_x] z_{xxxx} dx \\
&= \frac{2\eta}{q-1}\mathcal{N}(t) + 2(-\alpha(t))^{\frac{2}{q-1}} \int_0^1 z_{xxxx}^2 dx - 2(-\alpha(t))^{\frac{2}{q-1}} \int_0^1 z_{xxxxx}^2 dx \\
&\quad -2(-\alpha(t))^{\frac{q+1}{q-1}} \int_0^1 g(z_x) z_{xxxxx} dx. \qquad (4.4)
\end{aligned}$$

Therefore, by using Schwarz's inequality, Young's inequality and hypothesis (H2), we get

$$\begin{aligned}
&-2(-\alpha(t))^{\frac{q+1}{q-1}} \int_0^1 g(z_x) z_{xxxxx} dx \\
\leq\ & 2(-\alpha(t))^{\frac{q+1}{q-1}} \int_0^1 |g(z_x) z_{xxxxx}| dx \\
\leq\ & \frac{1}{2}(-\alpha(t))^{\frac{2q}{q-1}} \int_0^1 |g(z_x)|^2 dx + 2(-\alpha(t))^{\frac{2}{q-1}} \int_0^1 z_{xxxxx}^2 dx \\
\leq\ & \frac{\beta^2}{2}(-\alpha(t))^{\frac{2q}{q-1}} \int_0^1 |z_x|^{2q} dx + 2(-\alpha(t))^{\frac{2q}{q-1}} \int_0^1 z_{xxxxx}^2 dx. \qquad (4.5)
\end{aligned}$$

Now, we using the Sobolev embedding theorem, we get

$$\int_0^1 |z_{xx}|^{2q} dx \leq C_*^{2q}\left(\int_0^1 |z_{xx}|^2 dx + \int_0^1 |z_{xxxx}|^2 dx\right)^q. \qquad (4.6)$$

Here used the inequality

$$(a+b)^p \leq 2^{p-1}(a^p + b^p),$$

for $a, b \geq 0$ and $1 \leq p < \infty$. Substituting (4.5) and (4.6) into (4.4), we get

$$\begin{aligned} \mathcal{N}'(t) &\leq \frac{2\eta}{q-1}\mathcal{N}(t) + \frac{\beta^2 C_*^{2q}}{2}\left(-\alpha(t)\right)^{\frac{2q}{q-1}}\left(\int_0^1 |z_{xx}|^2\,dx + \int_0^1 |z_{xxxx}|^2\,dx\right)^q \\ &\leq \frac{2\eta}{q-1}\mathcal{N}(t) + \frac{\beta^2 C_*^{2q}}{2}\mathcal{N}(t)^q. \end{aligned} \tag{4.7}$$

Therefore, we obtain to consider inequality (4.7) Case 1 and Case 2.

Case 1: $\eta = 0$. Integrate (4.7) over $(0, 1)$, we get

$$\mathcal{N}(t)^{1-q} \geq \mathcal{N}(0)^{1-q} - \frac{(q-1)\beta^2 C_*^{2q}}{2}t. \tag{4.8}$$

Case 2: $\eta > 0$. Hence, integrating (4.8) with respect to t, we have

$$\int_{\mathcal{N}(0)}^{\mathcal{N}(t)} \frac{d\tau}{\frac{2\eta}{q-1}\tau + \frac{\beta^2 C_*^{2q}}{2}\tau^q} \leq t, \tag{4.9}$$

or through $\tau = \zeta^{\frac{1}{q}}$,

$$\frac{1}{q-1}\int_{\mathcal{N}(0)^{q-1}}^{\mathcal{N}(t)^{q-1}} \frac{d\zeta}{\zeta\left(\frac{2\eta}{q-1} + \frac{\beta^2 C_*^{2q}}{2}\zeta\right)} \leq t. \tag{4.10}$$

Furthermore, (4.10) is integrable and leads to

$$\mathcal{N}(t)^{1-q} \geq e^{-2\eta t}\left[\mathcal{N}(0)^{1-q} + \frac{(q-1)\beta^2 C_*^{2q}}{4\eta}\right] - \frac{(q-1)\beta^2 C_*^{2q}}{4\eta}.$$

Finally, utilizing Theorem 3, there exists a T^* such that $\lim_{t\to T^{*-}} \mathcal{N}(t) = 0$, leads to the lower bound

$$T^* \geq \frac{1}{2\eta}\ln\left(\frac{4\eta\mathcal{N}(0)^{1-q}}{(q-1)\beta^2 C_*^{2q}}\right),$$

which complete the proof of Theorem 4.1. □

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