The Aligarh Bulletin of Mathematics
Volume 42, Number 1 (2023), 93-105

ISSN: 0304-9787


# A projection computational technique for the solution Volterra-Fredholm integro-differential equations

**A. A. Victor**[1], **M. O. Etuk** [2], **C. Y. Ishola**[3], **A. O. Oladapo**[4]
**M. O. Ajisope** [5], **M. T. Raji** [6]

[1]Department of Mathematics, University of Ilorin, Nigeria
[2]Department of Mathematics and Statistics, Federal Polytechnic Bida
Niger State, Nigeria
[3]Department of Mathematics, National Open University of Nigeria
Jabi Abuja, Nigeria
[4]Department of Mathematics, Osun State University, Oshogbo, Osun state
Nigeria
[5]Department of Mathematics, Federal University Oye-ekiti
Ekiti State, Nigeria
[6]Department of Mathematics, Federal University of Agriculture, Abeokuta
Ogun State, Nigeria
Email: adekeyebisi5@gmail.com

*(Received: April 24, 2023, Accepted: May 4, 2023)*

**Abstract**

This work presents a projection computational technique for solving Volterra-Fredholm integro-differential equations (VFIDEs) via second kind Chebyshev polynomials as basis functions. The method transform VFIDEs into system of linear algebraic equations with the unknown Chebyshev coefficients, which is then solved using matrix inversion. To test for the accuracy and efficiency of the scheme, numerical example were solved and results obtained shows the method perform excellent than the compared results in the literature.

**Keywords and phrases:** Second kind Chebyshev polynomials, Volterra-Fredholm integro-differential equations, approximate solution.

**2020 AMS Subject Classification:** 65L60, 41A50

# 1 Introduction

This work take into consideration the VFIDE equation given in the form:

$$\sum_{i=0}^{n} \mu_i(z)v^i\left(z\right) = f\left(z\right) + \lambda_1 \int_{-1}^{z} k_1\left(z,t\right)v\left(t\right)dt + \lambda_2 \int_{-1}^{1} k_2\left(z,t\right)v\left(t\right)dt \tag{1.1}$$

With the initial conditions

$$v^i\left(0\right) = v_i,\ i = 0,1,2,\cdots n-1 \tag{1.2}$$

Where $k_1\left(z,t\right)$ and $k_2\left(z,t\right)$ and $\mu_i\left(z\right),\ \ i = 0,1,2,\cdots n$ with $\mu_n\left(z\right) \neq 0$ are known functions on the an interval $-1 \leq z \leq t \leq 1$. $\mu_1$, $\mu_2$,$\cdots$, $\mu_n$ , $\lambda_1$ , $\lambda_2$, are constant values, $f\left(z\right)$ is a known function and $v\left(z\right)$ is the unknown function to be determined. Integro-differential equations are widely employed as mathematical models in different fields. Integro-differential equations (IDEs) have attracted a lot of interest lately. Since many IDEs cannot be solved analytically, it would be quite helpful to establish exact approximations using numerical techniques. Here are only a few authors who have provided numerical methods for solving the IDEs: A technique for computing the VFIDEs [1], Utilizing an operational matrix and block-pulse functions, a direct approach for solving the first kind of Volterra integral equation is presented [2], Block pulse functions and operational matrices are used to numerically solve the VFIDEs [3], Chebyshev polynomial approach for the linear Fredholm-Volterra IDEs in their generic form [4,5,6], Fourth-order IDEs are treated using a variational iteration approach [7], and high-order nonlinear VFIDEs are treated using the differential transform method [8], Fixed point methods and Schauder bases are used to approximate the solution of the first-order mixed Fredholm-Volterra IDEs [9], and Chebyshev wavelets approximate the analytical solution for high-order IDEs [10], Numerical solution of VFIDEs using Legendre collocation method [11], Taylor collocation method and convergence analysis for the VFIDEs [12] and the numerical solution of high-order linear with variable coefficients using two proposed schemes for rational Chebyshev functions [13], For the solution of the linear and nonlinear Fredholm-Volterra IDEs [14,15], Bernstein polynomials are used as the basis function. Collocation method [16,17,18] for solving the VFIDEs. For the solution of Volterra IDEs [19] used the Legendre spectral element approach and VFIDEs are solved using the Lagrange collocation method in [20]. Other similar approach can be found in [21-27]. Inspired by the aforementioned studies, we provide a projection computational technique for the class of problem in equation (1) with improved accuracy and less rigorous work.

# 2 Materials and Methods

## 2.1 Chebyshev Polynomials of the Second Kind

The Chebyshev Polynomials of the second kind are defined by
$Q_n\left(z\right)=\frac{\sin[(n+1)cos^{-1}z]}{sin(cos^{-1}z)}; n=0,1,2,\ldots$ with $Q_0\left(z\right)=1$ and $Q_1\left(x\right)=2z$.

These polynomials form an orthogonal system with weight function $w\left(z\right)=\sqrt{1-z^2}$ on interval $[-1,1]$.
The recurrence relation is given by

$$Q_{n+1}\left(z\right)=2zQ_n\left(z\right)-Q_{n-1}\left(z\right), n=1,2,\ldots$$

With initial $Q_0\left(z\right)=1$, $Q_1\left(z\right)=2z$
Hence, the first few second kind Chebyshev Polynomials is given below

$$Q_0\left(z\right)=1,\ Q_1\left(z\right)=2z, Q_2\left(z\right)=4z^2-1,\ Q_3\left(z\right)=8z^3-4z.$$

"The shifted equivalent of it, denoted as ${Q_n}^*\left(z\right)$ , that valid in $\in[0,1]$ are given as: ${Q_n}^*\left(z\right)=Q_n\left(2z-1\right)$, $n=2,3,\cdots$ with initial ${Q_0}^*\left(z\right)=1$, ${Q_1}^*\left(z\right)=4x-2$."

## 2.2 Absolute error

We define absolute error as follows in this study: Absolute Error= $\left|V\left(z\right)-v\left(z\right)\right|$ ; $-1\leq z\leq 1$, where V$\left(z\right)$ is the exact solution and $v\left(z\right)$ is the approximate solution.

## 2.3 Proposed method

The work assumed an approximate solution by means of the second kind Chebyshev polynomials in the form:

$$v\left(z\right)=\sum_{i=0}^{n}Q_i\left(z\right)a_i \tag{2.1}$$

The unknown constants to be determined are $a_i$ , $i=0,1,\cdots n$.

Thus, substituting (2.1) into (1.1) gives $\sum_{i=0}^{n} \mu_i (z) {Q_i}^i (z) a_i$
$= f(z) + \lambda_1 \int_{-1}^{z} k_1 (z,t) \sum_{i=0}^{n} Q_i (t) a_i dt + \lambda_2 \int_{-1}^{1} k_2 (z,t) \sum_{i=0}^{n} Q_i (t) a_i dt$

Let

$$\zeta(z) = \sum_{i=0}^{n} \mu_i (z) {Q_i}^i (z) a_i , \eta(t) = \lambda_1 \int_{-1}^{z} k_1 (z,t) \sum_{i=0}^{n} Q_i (t) a_i dt$$

and

$$\tau(z) = \lambda_2 \int_{-1}^{1} k_2 (z,t) \sum_{i=0}^{n} Q_i (t) a_i dt \, . \tag{2.2}$$

Thus, (2.2) becomes

$$\zeta(z) - \eta(z) - \zeta(z) - \tau(z) = f(z) \tag{2.3}$$

The linear algebraic system of equations in (n+1) unknown constants $a'_i s$ are obtained by Collocating (2.3) at the evenly spaced point $z_i = a + \frac{(b-a)i}{n}$ , $(i = 0\,(1)\,n)$.

Additional equations are obtained from Eq. (1.2), which are represented in matrix form:

$$\begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & \cdots & W_{1n} \\ W_{21} & W_{22} & W_{23} & W_{24} & \cdots & W_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_{m1} & W_{m2} & W_{m3} & W_{m4} & \dots & W_{mn} \\ {W_{11}}^0 & {W_{12}}^0 & {W_{13}}^0 & {W_{14}}^0 & \dots & {W_{1n}}^0 \\ {W_{21}}^1 & {W_{22}}^1 & {W_{23}}^1 & {W_{24}}^1 & \dots & {W_{2n}}^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ {W_{m1}}^{n-1} & {W_{m2}}^{n-1} & {W_{m3}}^{n-1} & {W_{m4}}^{n-1} & \dots & {W_{mn}}^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} X_{11} \\ X_{21} \\ \vdots \\ \vdots \\ X_{mn} \\ {X_{11}}^0 \\ {X_{22}}^1 \\ \vdots \\ \vdots \\ {X_{mn}}^{n-1} \end{pmatrix} \tag{2.4}$$

where $W'_i s$ and $W_i^{0'} s$ are the coefficients of $a'_i s$ given as:

$$\mathrm{W_{11}, W_{12}, W_{13},\cdots W_{1n}}{=}\mu_2\mathrm{Q}^{''}\left(\mathrm{z_1}\right)+\mu_1\mathrm{Q}^{'}\left(\mathrm{z_1}\right)+\zeta\left(\mathrm{z_1}\right)+\mu_0\mathrm{Q}\left(\mathrm{z}\right)-\eta\left(\mathrm{z_1}\right)-\tau\left(\mathrm{z_1}\right)$$
$$\mathrm{W_{21}, W_{22}, W_{23},\cdots W_{2n}}{=}\mu_2\mathrm{Q}^{''}\left(\mathrm{z_2}\right)+\mu_1\mathrm{Q}^{'}\left(\mathrm{z}\right)+\zeta\left(\mathrm{z_2}\right)+\mu_0\mathrm{Q}\left(\mathrm{z_2}\right)-\eta\left(\mathrm{z_2}\right)-\tau\left(\mathrm{z_2}\right)$$
$$\mathrm{W_{31}, W_{32}, W_{33},\cdots W_{3n}}{=}\mu_2\mathrm{Q}^{''}\left(\mathrm{z_3}\right)+\mu_1\mathrm{Q}^{'}\left(\mathrm{z_3}\right)+\zeta\left(\mathrm{z_3}\right)+\mu_0\mathrm{Q}\left(z\right)-\eta\left(\mathrm{z_3}\right)-\tau\left(z\right)$$

$\mathrm{W_{11}}^0, \mathrm{W_{12}}^0, \mathrm{W_{13}}^0,\cdots \mathrm{W_{1n}}^0$ are values of $Q_i$ and $X_{is}$ are values of $f(z_i)$.

Let equation (2.4) be:

$$G\left(z_i\right)A = B(z_i) \tag{2.5}$$

Where $G\left(z_i\right) = \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & \cdots & W_{1n} \\ W_{21} & W_{22} & W_{23} & W_{24} & \cdots & W_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_{m1} & W_{m2} & W_{m3} & W_{m4} & \ldots & W_{mn} \\ {W_{11}}^0 & {W_{12}}^0 & {W_{13}}^0 & {W_{14}}^0 & \ldots & {W_{1n}}^0 \\ {W_{21}}^1 & {W_{22}}^1 & {W_{23}}^1 & {W_{24}}^1 & \ldots & {W_{2n}}^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ {W_{mn}}^{n-1} & {W_{m2}}^{n-1} & {W_{m3}}^{n-1} & {W_{m4}}^{n-1} & \ldots & {W_{mn}}^{n-1} \end{pmatrix}$,

$$A = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_n \end{pmatrix}, B\left(z_i\right) = \begin{pmatrix} X_{11} \\ X_{21} \\ \vdots \\ \vdots \\ X_{mn} \\ {X_{11}}^0 \\ {X_{22}}^1 \\ \vdots \\ \vdots \\ {X_{mn}}^{n-1} \end{pmatrix}$$

Multiply both sides of equation (2.5) by $G\left(z_i\right)^{-1}$ gives

$$A = G\left(z_i\right)^{-1} B(z_i) \tag{2.6}$$

The required approximate solution is obtained by solving Eq.(2.6) and then substituting the unknown constant values into the assumed approximate solution.

# 3 Numerical Examples

**Example 3.1** [6]:Consider the Volterra integro-differential equation of second order

$$v^{''}\left(z\right) + zv^{'}\left(z\right) - zv\left(z\right) = e^{x} - (z+1)\text{sin}z\ - z\text{sin}z\ + \int_{-1}^{z} \sin(z)\, e^{-t} v\left(t\right) dt$$

subject to the conditions

$$v\left(0\right) = 1, v^{'}\left(0\right) = 1.$$

The exact solution is

$$v\left(z\right) = e^{z}.$$

Using the method outlined above, we obtained the following unknown constants:

$a_0 = 1.13031935229582, a_1 = 0.542990644215075, a_2 = 0.133010595011089,$
$a_3 = 0.0218970380731072, a_4 = 0.00271434567755696, a_5 = 0.000269842693362447,$
$a_6 = 0.0000227798900969393, a_7 = 0.00000161974692477898.$

Thus, the approximate solution is given as;

$$v\left(z\right) = 1.000000323 + 0.9999992337z + 1.5000169493z^2 + 0.1666709182z^3$$
$$+\ 0.04160713964z^4 + 0.008323974779z^5 + 0.001457912966z^6 + 0.0002073276064z^7.$$

**Example 3.2** [6]. Let us consider the problem 3.3 $v^{''}\left(z\right) + zv^{'(x)} - 2v\left(z\right) = z\text{cos}z\ - 3\text{sin}z$

Subject to the conditions $v\left(0\right) = 0, v^{'(0)} = 1$. The exact solution is $v\left(z\right) = \text{sin}z$ Using the method outlined above, we obtained the following unknown constants:

$$a_0 = -2.05030655677534 \times 10^{-15}, a_1 = 0.459614039666964,$$
$$a_2 = -3.98522913460800 \times 10^{-16}, a_3 = -7.99858803824260 \times 10^{-18},$$
$$a_4 = 0.000251259077919744,\ a_5 = 0.000251136330810642,$$
$$a_6 = 1.32319620694249 \times 10^{-18}, c_7 = -0.0000149131183219825,$$

Consequently, the approximate solution is given as:

$$v\left(z\right) = -1.661105428 \times 10^{-15} + z - 1.466351889 \times 10^{-15} z^2 - 0.1666644670 z^3$$
$$- 2.33833105\,2 \times 10^{-15} z^4 + 0.008326622365 z^5 + 8.468455725 \times 10^{-17} z^6$$
$$- 0.0001908879145 x^7$$

**Example 3.3** [4, 6 ]**:** Consider the following fifth-order

Fredholm integro- differential equation

$$v^{(v)}\left(z\right) - x^2 v^{'''} z - V^{'(z)} - zv\left(z\right) = z^2 \cos z\ - z \sin z + \int_{-1}^{1} v\left(t\right) dt$$

subject to the conditions

$$v\left(0\right) = 0, v^{'(0)} = 1, v^{''}\left(0\right) = 0, v^{'''}\left(x\right) = -1, v^{(iv)}\left(0\right) = -1.$$

The exact solution is $v\left(z\right) = \sin z$. Using the method outlined above, we obtained the following unknown constants:

$$a_0 = -2.77555756156289 \times 10^{-15}, a_1 = 0.459613929931025,$$
$$a_2 = 5.89805981832114 \times 10^{-17}, a_3 = -0.0198131174943388,$$
$$a_4 = 1.35525271560688 \times 10^{-19}, a_5 = 0.000251260108178906,$$
$$a_6 = 8.47032947254300 \times 10^{-21}, a_7 = -0.00000150726946032969,$$
$$a_8 = 3.17637355220363 \times 10^{22}, a_9 = 5.13558377666354 \times 10^{-9}.$$

Consequently, the approximate solution is given as:

$v\left(z\right) = z + 6.844026214 \times 10^{-19} z^6 + 1.414545023 \times 10^{-18} z^4 + 2.345120828 \times 10^{-16} z^2 - 2.834411423 \times 10^{-15} + 0.000002629418894 z^9 - 8.131516293 \times 10^{-20} z^8 - 0.0001981893287 z^7 + 0.008333170310 z^5 - 0.1666666667.$

## 4 Results and Discussion

**Table 1 Shows comparison of the absolute errors for the Example 3.1**

| z | [6]<br>N=9 | Our Method<br>n=7 |
|---|---|---|
| -1.0 | 2.0697374 E-2 | 1.429E-06 |
| -0.8 | 1.1470384 E-2 | 6.971E-07 |
| -0.6 | 4.420981E-3 | 1.560E-06 |
| -0.4 | 9.73181E-4 | 1.894E-6 |
| -0.2 | 5.9298E-5 | 1.032E-7 |
| 0 | 1.0E-8 | 3.230E-7 |
| 0.2 | 8.849E-5 | 7.881E-7 |
| 0.4 | 1.21776E-3 | 1.661E-6 |
| 0.6 | 5.26806E-3 | 1.465E-8 |
| 0.8 | 1.334832E-2 | 6.871E-7 |
| 1.0 | 2.336246E-2 | 1.951E-6 |

**Table 2 Shows comparison of the absolute errors for the Example 3.2**

| z | [6]<br>N=9 | Our Method<br>n=7 |
|---|---|---|
| -1.0 | 2.0697374 E-2 | 2.827E-07 |
| -0.8 | 1.1470384 E-2 | 1.375E-07 |
| -0.6 | 4.420981E-3 | 1.362E-07 |
| -0.4 | 9.73181E-4 | 8.367E-08 |
| -0.2 | 5.9298E-5 | 1.554E-05 |
| 0 | 1.0E-8 | 0.000 |
| 0.2 | 8.849E-5 | 1.554E-08 |
| 0.4 | 1.21776E-3 | 8.367E-08 |
| 0.6 | 5.26806E-3 | 1.362E-07 |
| 0.8 | 1.334832E-2 | 1.375E-07 |
| 1.0 | 2.336246E-2 | 2.827E-07 |

**Table 3 Shows comparison of the absolute errors for the Example 3.3**

| z | [4] | [6] N=9 | Our Method n=9 |
|---|---|---|---|
| -1.0 | 1.3591E-5 | 9.0E-8 | 4.150E-08 |
| -0.8 | 3.1940E-6 | 3.9E-8 | 2.140E-08 |
| -0.6 | 5.3450E-7 | 1.4E-8 | 7.620E-09 |
| -0.4 | 4.8962E-8 | 4.0E-9 | 1.330E-09 |
| -0.2 | 1.0561E-9 | 1.0E-9 | 5.600E-11 |
| 0 | 0.00000 | 0.0000 | 4.800E-12 |
| 0.2 | 5.1234E-10 | 1.0E-9 | 5.600E-11 |
| 0.4 | 1.1835E-8 | 1.6E-8 | 1.330E-09 |
| 0.6 | 6.7471E-8 | 1.79E-7 | 7.620E-09 |
| 0.8 | 2.2275E-7 | 1.156E-6 | 2.140E-08 |
| 1.0 | 5.5371E-7 | 5.292E-6 | 4.110E-08 |

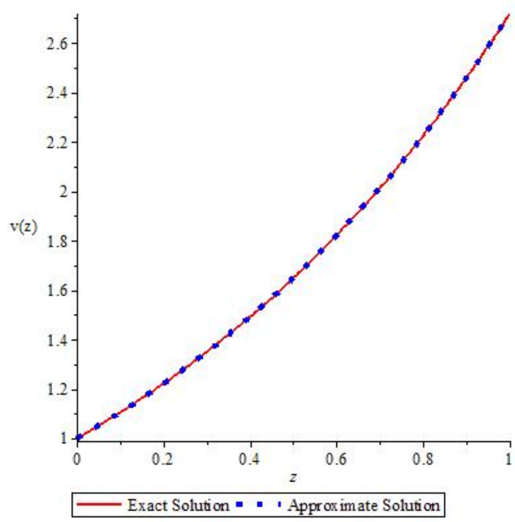


Figure 1: The exact solution and approximation of the example 3.1 problem's solution in graphical form

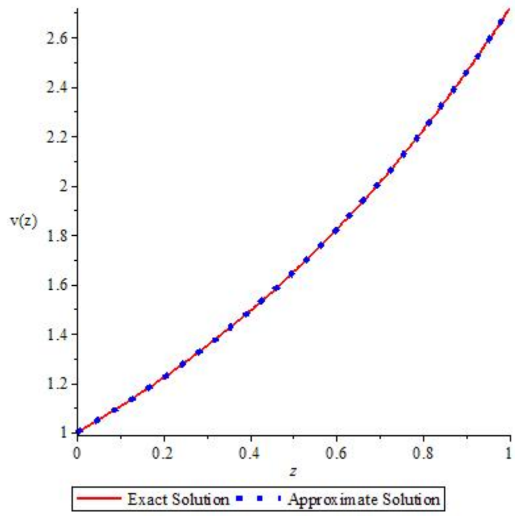


Figure 2: The exact solution and approximation of the Example 3.2 problem's solution in graphical form

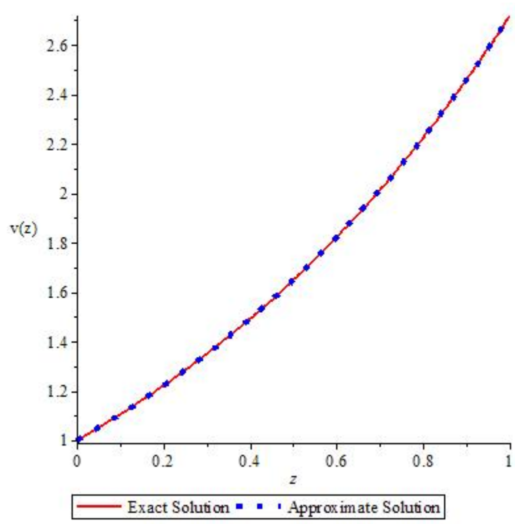


Figure 3: The exact solution and approximation of the Example 3.3 problem's solution in graphical form

# 5 Conclusion

In this study, the proposed scheme has been successfully applied to get numerical solutions to VFIDEs by using second-kind Chebyshev polynomials. Numerical examples are provided to illustrate the technique's accuracy and efficiency using tables and figures. The error table found in Table 1-3 shows that the method used was more accurate because the errors are smaller than those found in [4, 6], and the graphs of the approximation solutions in Figure 1–3 show excellent agreement with the exact solutions. On the basis of this work, researchers can use this technique for a variety of additional VFIDEs.

# References


[1] K. Maleknejad , M. R. Fadaei Yami, A computational method for system of Volterra Fredholm integral equations. Applied Mathematics Computation, 183(1), (2006), 589-595.

[2] E. Babolian, Z. Masouri, Direct method to solve Volterra integral equation of the first kind using operational matrix with block-pulse functions. Journal of Computational and Applied Mathematics., 220(1) (2008) 51-57. https://doi.org/10.1016/j.cam.2007.07.029

[3] L. Rahmani , B. Rahimi, M. Mordad, Numerical Solution of Volterra-Fredholm Integro-Differential Equation by Block Pulse Functions and Operational Matrices. Gen. Math. Notes, 4(2) (2011) 37-48.

[4] A. Akyüz, Chebyshev polynomial approach for linear Fredholm-Volterra integro-differential equations in the most general form, Applied Mathematics Computation, 181(1) (2006):103-112.doi. 10. 1016/j.amc.2006.01.018.

[5] H. M. El-Hawary, T. S. El-Sheshtawy, Spectral method for solving the general form linear Fredholm–Volterra integro differential equations based on Chebyshev polynomials. J Mod Met Numer Math. 1 (2010) 1–11.

[6] G. Yüksel, M. Gülsu, M. Sezer, A Chebyshev polynomial approach for high-order linear Fredholm-Volterra integro-differential equations, Gazi University Journal of Science 25(2) (2012) 393-401.

[7] N. H. Sweilam, Fourth order integro-differential equations using variational iteration method, Computer Mathematics Applications, 54 (2007) 1086-1091.

[8] S. H. Behiry, S. I. Mohamed, Solving high-order nonlinear Volterra-Fredholm integro-differential equations by differential transform method,.Natural Science 4(8) (2012) 581-587. doi.org/10.4236/ns.2012.48077.

[9] M. I. Berenguer, D. Gamez, A. J. L´ Opez Linares, Fixed-point iterative algorithm for the linear Fredholm-Volterra integro-differential equation, Journal of Computational and Applied Mathematics, Article ID 370894 (2012) 12 pages doi:10.1155/2012/370894.

[10] H. Aminikhah , S. Hosseini, J. Alavi, Approximare analytical solution for high-order integro-differential equation by Chebyshev waveletes, Information Sciences Letters. 4(14) (2015) 31-39.

[11] S. Nemati. (2014) Numerical solution of Volterra–Fredholm integral equations using Legendre collocation method. Journal of Computational and Applied Mathematics S0377-0427(14) 00437-3, 1-16. http://dx.doi.org/10.1016/j.cam.2014.09.030 .

[12] K. Wang, Q. Wang, Taylor collocation method and convergence analysis for the Volterra-Fredholm integral equations, Journal of Computational and Applied Mathematics 260 (2013) 294 -300.

[13] M. Ramadan, K. Raslan, A. Hadhoud, M. Nassar, Numerical solution of high-order linear integro-differential equations with variable coefficients using two proposed schemes for rational Chebyshev functions, New Trends in Mathematical Sciences, 4(3) (2016) 22–35. 10. 20852/ntmsci.2016318802.

[14] O. R. Isık, M. Sezer, Z. G¨ uney, Bernstein series solution of a class of linear integro-differential equations with weakly singular kernel, Applied Mathematics and Computation, 217(16) (2011) 7009–7020.

[15] A. Akyüz-DaGcJoLlu, N. Acar, C. Güler, Bernstein collocation method for solving nonlinear Fredholm-Volterra integro differential equations in the most general form, Journal of Applied Mathematics , Article ID 134272, (2014)1-8 pages. doi.org/10.1155/2014/134272

[16] N. Ebrahimi, J. Rashidinia , Spline collocation for Fredholm and Volterra integro-differential equations, International Journal of Mathematical Model and Computation, 4 (2014) 289–298.

[17] O. K. Kürkçü, E. Aslan, M. Sezer, A novel collocation method based on residual error analysis for solving integro-differential equations using hybrid Dickson and Taylor polynomials, Sains Malaysiana, 46, (2017) 335–347.

[18] V. N. Mishra, H. R. Marasi, H. Shabanian M. N. Sahlan, Solution of Voltra–Fredholm integro-differential equations using Chebyshev collocation method. Global Journal Technology and Optimization 2017; (2017) 1-4. doi:10.4172/2229-8711.1000210.

[19] M. Lotfi, A. Alipanah, Legendre spectral element method for solving Volterra-integro differential equations. Results in Applied Mathematics, 7 (2020) 1-11.. doi:10.1016/j.rinam.2020.100116.

[20] K. Y. Wang, Q. S. Wang, Lagrange collocation method for solving VolterraFredholm integral equations, Applied Mathematics Computation 219 (2013) 10434–10440.

[21] T. Oyedepo, A. F. Adebisi, R. M. Tayo, J. A. Adedeji, M. A. Ayinde, O. J. Peter. Perturbed least squares technique for solving volterra fractional integro-differential equations based on constructed orthogonal polynomials. Journal of Mathematical and Computer Science. 11(2021), 203-218.

[22] Oyedepo, T., Uwaheren, O. A., Okperhie E. P. and O. J. Peter. Solution of Fractional Integro-Differential Equation Using Modified Homotopy Perturbation Technique and Constructed Orthogonal Polynomials as Basis Functions. Journal of Science Technology and Education 7, 157-164 (2019).

[23] C. Y. Ishola, O. A. Taiwo. A. F, Adebisi, O. J. Peter. Numerical solution of two-dimensional Fredholm integro-differential equations by Chebyshev integral operational matrix method. Journal of Applied Mathematics and Computational Mechanics. 21(1), 29-40, (2022).

[24] O. A. Uwaheren, A. F. Adebisi, C. Y. Ishola, M. T. Raji, A. O. Yekeem and O. J. Peter, "Numerical Solution of Volterra integro-differential Equations by Akbari-Ganji's Method", BAREKENG: J. Math. & App., 16(3): 1123-1130, (2022).

[25] A. F. Adebisi, T. A. Ojurongbe, K. A. Okunlola, O. J. Peter. Application of Chebyshev polynomial basis function on the solution of volterra integro-differential equations using Galerkin method Mathematics and Computational Sciences, 2(1), 2021: 41-51.

[26] A. F. Adebisi, K. A. Okunola, M. T. Raji, J. A. Adedeji1 & O. J. Peter. Galerkin and perturbed collocation methods for solving a class of linear fractional integro-differential equations. The Aligarh Bulletin of Mathematics. 40(2) (2021), 45-57.

[27] A. F. Adebisi, O. A. Uwaheren, O. E. Abolarin, R. M. Tayo, J. A. Adedeji, O. J. Peter. Solution of Typhoid Fever Model by Adomian Decomposition Method. Journal of Mathematical and Computer Science.11(2021), 2, 1242-1255.