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A note on two-sided ideals with derivations in prime rings

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Abstract

For fixed positive integers m and n, $\mathfrak{S}$ be a prime ring with $\mathrm{char}(\mathfrak{S}) \neq 2$ with the Martindale quotient ring $\mathcal{Q}$. $\mathfrak{L}(\neq 0)$ and $\mathfrak{F}(\neq 0)$ be an ideal and derivation of $\mathfrak{S}$, respectively. If $[\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m]^n = [\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m] \ \forall \ \mathfrak{x}_1, \mathfrak{x}_2 \in \mathfrak{L}$, then $\mathfrak{S}$ is commutative.

1 Introduction

Unless otherwise mentioned, $\mathfrak{S}$ will be an associative ring throughout this article. The centre of $\mathfrak{S}$ is denoted by $Z(\mathfrak{S})$, $\mathcal{U}$ is used for Utumi quotient ring, $\mathcal{Q}$ is Martindale quotient ring and $\mathfrak{C}$ is used for extended centroid. In [2] contains the axiomatic formulations, definitions and attributes of these quotient rings. It's worth noting that $\mathfrak{C}$ is a field while $\mathcal{Q}$ is a prime ring with identity. For $\mathfrak{x}_1, \mathfrak{x}_2 \in \mathfrak{S}$ and each $n \geq 0$, set $[\mathfrak{x}_1, \mathfrak{x}_2]_0 = \mathfrak{x}_1$, $[\mathfrak{x}_1, \mathfrak{x}_2]_1 = \mathfrak{x}_1\mathfrak{x}_2 - \mathfrak{x}_2\mathfrak{x}_1$, for non-commuting indeterminate and for $m = 1, 2, \cdots$ the Engel condition is then a polynomial $[\mathfrak{x}_1, \mathfrak{x}_2]_m = [[\mathfrak{x}_1, \mathfrak{x}_2]_{m-1}, \mathfrak{x}_2]$. If there

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is a positive integer m satisfying $[\mathfrak{x}_1, \mathfrak{x}_2]_m = 0$, then the ring $\mathfrak{S}$ satisfies an Engel condition. If $a\mathfrak{S}b = (0) \implies a = 0$ or $b = 0\ \forall\ a, b \in \mathfrak{S}$, then $\mathfrak{S}$ is called a prime ring and if $a\mathfrak{S}a = (0) \implies a = 0\ \forall\ a \in \mathfrak{S}$, then it is semiprime ring. An additive mapping $\mathfrak{F} : \mathfrak{S} \to \mathfrak{S}$ is known as derivation if $\mathfrak{F}(\mathfrak{x}_1\mathfrak{x}_2) = \mathfrak{F}(\mathfrak{x}_1)\mathfrak{x}_2 + \mathfrak{x}_1\mathfrak{F}(\mathfrak{x}_2)\ \forall\ \mathfrak{x}_1, \mathfrak{x}_2 \in \mathfrak{S}$. In particular, if $\mathfrak{F}_a = [a, \mathfrak{x}_1]\ \forall\ \mathfrak{x}_1 \in \mathfrak{S}$ then $\mathfrak{F}$ is called an inner derivation generated by an element $a \in \mathfrak{S}$. In addition, if the extension of $\mathfrak{F}$ to $\mathcal{Q}$ is inner, $\mathfrak{F}$ is called $\mathcal{Q}$-inner, otherwise it is $\mathcal{Q}$-outer.

Jacobson [10] established a classic ring theory result that generalizes both Wedderburn's theorem that any finite division ring is commutative as well as the fact that any Boolean ring is commutative. We are referring to the following theorem: Any ring in which $\mathfrak{x}_1^n = \mathfrak{x}_1$, $n > 1$ is a fixed integer, is necessarily commutative. Moreover, Herstein [8] demonstrated that a ring must be commutative if it satisfied $[\mathfrak{x}_1, \mathfrak{x}_2]^n = [\mathfrak{x}_1, \mathfrak{x}_2]$ for every $\mathfrak{x}_1, \mathfrak{x}_2 \in \mathfrak{S}$, $n > 1$ is a fixed integer. In 2011, Huang [9] addressed prime ring commutativity and showed that if a prime ring $\mathfrak{S}$ admits a derivation $\mathfrak{F}$ such that $\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2])^m = [\mathfrak{x}_1, \mathfrak{x}_2]_n\ \forall\ \mathfrak{x}_1, \mathfrak{x}_2 \in \mathfrak{L}$ the nonzero ideal of $\mathfrak{S}$, where $m, n > 1$ are integers, must be commutative. Giambruno et al. [7], on the other hand, generalized Herstein's conclusion for Engel type conditions and found that a ring must be commutative if it satisfies $([\mathfrak{x}_1, \mathfrak{x}_2]_m)^n = [\mathfrak{x}_1, \mathfrak{x}_2]_m$. In recent times, the author with Raza [16] received that a prime ring is commutative if it satisfies $\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m)^n = [\mathfrak{x}_1, \mathfrak{x}_2]_m\ \forall\ \mathfrak{x}_1, \mathfrak{x}_2 \in \mathfrak{L}$, a nonzero ideal of $\mathfrak{S}$. Encouraged by previous findings, we investigate semiprime and prime rings $\mathfrak{S}$ satisfying a derivation $\mathfrak{F}$ such that $[\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m]^n = [\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m]\ \forall\ \mathfrak{x}_1, \mathfrak{x}_2$ in some suitable subset of $\mathfrak{S}$.

2 The Results

We'll define certain notations and collect some well-known results before coming to our main theorem, which we'll used later.

Fact 2.1. *Let $\mathfrak{S}$ be a prime ring and $\mathfrak{L}$ a two sided ideal of $\mathfrak{S}$. Then $\mathfrak{L}$, $\mathfrak{S}$, $\mathcal{U}$ satisfy the same generalized polynomial identities with coefficients in $\mathcal{U}$ (see [4]).*

Fact 2.2. *Every derivation $\mathfrak{F}$ of $\mathfrak{S}$ can be uniquely extended to a derivation on $\mathcal{U}$ (see Proposition 2.5.1 of [2]).*

Fact 2.3. *Let $\mathfrak{S}$ be a prime ring and $\mathfrak{L}$ a two sided ideal of $\mathfrak{S}$. Then $\mathfrak{S}$, $\mathfrak{L}$ and $\mathcal{U}$ satisfy the same differential identities (see [14]).*

We are now going to prove our main theorem:

Theorem 2.1. *Let $\mathfrak{S}$ be a prime ring with char$(\mathfrak{S}) \neq 2$. $\mathfrak{L}(\neq 0)$ and $\mathfrak{F}(\neq 0)$ be an ideal and derivation of $\mathfrak{S}$, respectively. If $\mathfrak{L}$ satisfies $[\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m]^n = [\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m]$, then $\mathfrak{S}$ is commutative.*

Proof. We divided our proof into two cases in light of Kharchenko's theory [11]. Firstly, we suppose that $\mathfrak{F}$ is an $\mathcal{Q}$-inner derivation generate by an element $\lambda \in \mathcal{Q}$, i.e., $\mathfrak{F}(\mathfrak{x}_1) = [\lambda, \mathfrak{x}_1]$ $\forall$ $\mathfrak{x}_1 \in \mathfrak{S}$, we see that

$$[[\lambda, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m]^n = [[\lambda, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m] \ \forall \ \mathfrak{x}_1, \mathfrak{x}_2 \in \mathfrak{L}. \tag{2.1}$$

If $\lambda \in \mathfrak{C}$, we are done. We start with $\lambda \notin \mathfrak{C}$. In this case (2.1) is a non-trivial GPI by Chuang [4, Theorem 2] for $\mathcal{Q}$ as $\lambda \notin \mathfrak{C}$. As a result, by Martindale's [15], a dense ring of linear transformations of a vector space $\mathscr{V}$ over $\mathfrak{C}$ is isomorphic to a primitive ring $\mathcal{Q}$. First we assume that $dim_{\mathfrak{C}}(\mathscr{V}) = l$, where $l \geq 3$ a finite positive integer. Now we assume that $\mathfrak{M}_l(\mathfrak{C})$, the full matrix ring of all $l \times l$ matrices over $\mathfrak{C}$. As usual, by e_{ij}, $1 \leq i, \ j \leq l$, we denote the matrix unit whose (i, j)-entry is equal to 1 and all its other entries are equal to zero. Let $\lambda = \sum_{ij} \alpha_{ij} e_{ij}$. We need to show that λ is a diagonal matrix. Let $\mathfrak{x}_1 = e_{ij}$ and $\mathfrak{x}_2 = e_{jj}$. In this case we deduce that $([[\lambda, e_{ij}], e_{ij}])^n = [[\lambda, e_{ij}], e_{ij}]$ that is, $e_{ij}\lambda e_{ij} = 0$ and hence $\alpha_{ji} = 0$, for any $i \neq j$. As a result, λ is a diagonal matrix. We'll show that λ is a central matrix in the next step. Furthermore, if an automorphism θ of $\mathfrak{M}_l(\mathfrak{C})$ exists such that

$$([[\theta(\lambda), [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m])^n = [[\theta(\lambda), [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m],$$

is a GPI of $\mathfrak{S}$. This proved that $\theta(\lambda)$ is a diagonal matrix $\forall \ \theta \in Aut(\mathfrak{M}_l(\mathfrak{C}))$. In particular, let $r \neq t$ and

$$\theta(x) = (1 + e_{rt})x(1 - e_{rt}) = x + e_{rt}x - xe_{rt} - e_{rt}xe_{rt}.$$

Put $\theta(\lambda) = \sum \theta(\lambda)'_{ij} e_{ij}$ with $\theta(\lambda)'_{ij} \in \mathfrak{C}$. Since $\theta(\lambda)$ is diagonal, we have $\theta(\lambda)'_{lt} = 0$. Hence $\lambda_{tt} - \lambda_{ll} = 0$ and hence we arrives at conclusion that λ is a central matrix, a contradiction.

Now, suppose that $dim_{\mathfrak{C}}(\mathscr{V}) = \infty$. Then we have

$$[[\lambda, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]]^n = [[\lambda, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m] \ \forall \ \mathfrak{x}_1, \mathfrak{x}_2 \in \mathcal{Q}.$$

Furthermore, for any minimal idempotent element $e \in \mathfrak{H}$, Martindale's theorem [15] shows that $\mathfrak{S} = \mathfrak{S}\mathfrak{C}$ is a primitive ring with $soc(\mathcal{Q}) = \mathfrak{H} \neq 0$ and $e\mathfrak{H}e$ is a simple central algebra finite dimensional over $\mathfrak{C}$. We can also suppose that $\mathfrak{H}$ is non-commutative, as $\mathcal{Q}$ must be commutative otherwise. Obviously, $\mathfrak{H}$ fulfils. $[[\lambda, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]]^n = [[\lambda, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m]$. Given that $\mathfrak{H}$ is a simple ring, so either $\mathfrak{H}$ doesn't contain any non-trivial idempotent element or $\mathfrak{H}$ is obtained by its idempotents. In the latter case, assume that $\mathfrak{H}$ includes two minimal orthogonal idempotent components e and e' of rank 1, by the hypothesis, for $[\mathfrak{x}_1, \mathfrak{x}_2]_m = [e\mathfrak{x}_1, e']_m = e\mathfrak{x}_1 e'$, we obtain

$$e\mathfrak{x}_1 e'(\lambda) e\mathfrak{x}_1 e' = 0.$$

As a result, we receive $e'\lambda e\mathfrak{x}_1 e'\lambda e\mathfrak{x}_1 e'\lambda e = 0$. Since, $\mathfrak{S}$ is prime, we have $e'\lambda e = 0$. As e is of rank 1, we get $e\lambda(1-e) = 0$ and $(1-e)\lambda e = 0$ i.e., $e\lambda = e\lambda e = \lambda e$. Since $\mathfrak{H}$ is obtained by e and e', therefore, $[\lambda, e] = 0$ and $[\lambda, \mathfrak{H}] = 0$. Above discussion gives, either $\lambda \in \mathfrak{C}$ or $\mathfrak{S}$ is commutative.
In the other situation, we are done by contradiction.

Next, we consider that $\mathfrak{H}$ cannot contain two minimal orthogonal idempotent elements. So, $\mathfrak{H} = \mathfrak{G}$ for suitable division ring $\mathfrak{G}$ finite dimensional over its centre. This denotes $\mathfrak{G} = \mathfrak{H}$ and $\lambda \in \mathfrak{H}$. For $\mathfrak{K}$ be a field, by [17, Theorem 2.3.29] ([12, Lemma 2]) we have, $\mathfrak{H} \subseteq \mathfrak{M}_l(\mathfrak{K})$, where, $\mathfrak{M}_l(\mathfrak{K})$ fulfil $[[\lambda, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]]^n = [[\lambda, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m]$. We get $\mathfrak{H} \subseteq \mathfrak{K}$ if $m = 1$, which is contradiction. Further, if $m \geq 2$, then $\lambda \in Z(\mathfrak{M}_l(\mathfrak{K}))$, as we have shown recently.

Finally, consider if $\mathfrak{H}$ does not contain any non-trivial idempotent element, then $\mathfrak{H}$ is finite dimensional division ring i.e., $\mathfrak{H}$ is a field, and so $\mathfrak{S}$ is commutative as well. If $\mathfrak{H}$ is infinite, then $\mathfrak{H} \otimes_{\mathfrak{C}} \mathfrak{K} \cong \mathfrak{M}_l(\mathfrak{K})$, where $\mathfrak{K}$ is a splitting field of $\mathfrak{H}$. The previous argument say that $\lambda \in Z(\mathfrak{S})$.

Assume that $\mathfrak{F}$ is not $\mathcal{Q}$-inner. Note that

$$\begin{aligned} \mathfrak{F}([\mathfrak{x}_1,\mathfrak{x}_2]_m) &= \textstyle\sum_{k=1}^{m}(-1)^k \binom{m}{k} \left(\sum_{i+j=k-1} \mathfrak{x}_2^i\mathfrak{F}(\mathfrak{x}_2)\mathfrak{x}_2^j\right) \mathfrak{x}_1\mathfrak{x}_2^{m-k} \\ &+ \textstyle\sum_{k=0}^{m}(-1)^k \binom{m}{k} \mathfrak{x}_2^k\mathfrak{F}(\mathfrak{x}_1)\mathfrak{x}_2^{m-k} \\ &+ \textstyle\sum_{k=0}^{m-1}(-1)^m \binom{m}{k} \mathfrak{x}_2^k\mathfrak{x}_1 \left(\sum_{r+s=m-k-1} \mathfrak{x}_2^r\mathfrak{F}(\mathfrak{x}_2)\mathfrak{x}_2^s\right). \end{aligned} \tag{2.2}$$

In view of our hypothesis and well-known results, we can say that $\mathfrak{S}$ satisfies $[\mathfrak{F}([\mathfrak{x}_1,\mathfrak{x}_2]_m),[\mathfrak{x}_1,\mathfrak{x}_2]_m]^n = [\mathfrak{F}([\mathfrak{x}_1,\mathfrak{x}_2]_m),[\mathfrak{x}_1,\mathfrak{x}_2]_m]$ which is rewritten as

$$\begin{aligned} &\Bigg(\Bigg[\sum_{k=1}^{m}(-1)^k \binom{m}{k} \Bigg(\sum_{i+j=k-1} \mathfrak{x}_2^i\mathfrak{F}(\mathfrak{x}_2)\mathfrak{x}_2^j\Bigg) \mathfrak{x}_1\mathfrak{x}_2^{m-k} + \sum_{k=0}^{m}(-1)^k \binom{m}{k} \mathfrak{x}_2^k\mathfrak{F}(\mathfrak{x}_1)\mathfrak{x}_2^{m-k} \\ &+\sum_{k=0}^{m-1}(-1)^m \binom{m}{k} \mathfrak{x}_2^k\mathfrak{x}_1 \Bigg(\sum_{r+s=m-k-1} \mathfrak{x}_2^r\mathfrak{F}(\mathfrak{x}_2)\mathfrak{x}_2^s\Bigg), \sum_{k=0}^{m}(-1)^k \binom{m}{k} \mathfrak{x}_2^k\mathfrak{x}_1\mathfrak{x}_2^{m-k}\Bigg]\Bigg)^n \\ &= \Bigg[\sum_{k=1}^{m}(-1)^k \binom{m}{k} \Bigg(\sum_{i+j=k-1} \mathfrak{x}_2^i\mathfrak{F}(\mathfrak{x}_2)\mathfrak{x}_2^j\Bigg) \mathfrak{x}_1\mathfrak{x}_2^{m-k} + \sum_{k=0}^{m}(-1)^k \binom{m}{k} \mathfrak{x}_2^k\mathfrak{F}(\mathfrak{x}_1)\mathfrak{x}_2^{m-k} \\ &+\sum_{k=0}^{m-1}(-1)^m \binom{m}{k} \mathfrak{x}_2^k\mathfrak{x}_1 \Bigg(\sum_{r+s=m-k-1} \mathfrak{x}_2^r\mathfrak{F}(\mathfrak{x}_2)\mathfrak{x}_2^s\Bigg), \sum_{k=0}^{m}(-1)^k \binom{m}{k} \mathfrak{x}_2^k\mathfrak{x}_1\mathfrak{x}_2^{m-k}\Bigg]. \end{aligned}$$

In the light of Kharchenko's theorem [11], we find that

$$\begin{aligned} &\Bigg(\Bigg[\sum_{k=1}^{m}(-1)^k \binom{m}{k} \Bigg(\sum_{i+j=k-1} \mathfrak{x}_2^i\mathfrak{w}\mathfrak{x}_2^j\Bigg) \mathfrak{x}_1\mathfrak{x}_2^{m-k} + \sum_{k=0}^{m}(-1)^k \binom{m}{k} \mathfrak{x}_2^k\mathfrak{z}\mathfrak{x}_2^{m-k} \\ &+\sum_{k=0}^{m-1}(-1)^m \binom{m}{k} \mathfrak{x}_2^k\mathfrak{x}_1 \Bigg(\sum_{r+s=m-k-1} \mathfrak{x}_2^r\mathfrak{w}\mathfrak{x}_2^s\Bigg), \sum_{k=0}^{m}(-1)^k \binom{m}{k} \mathfrak{x}_2^k\mathfrak{x}_1\mathfrak{x}_2^{m-k}\Bigg]\Bigg)^n \end{aligned}$$

$$= \left[\sum_{k=1}^{m}(-1)^k \begin{pmatrix} m \\ k \end{pmatrix} \left(\sum_{i+j=k-1} \mathfrak{x}_2^i \mathfrak{w} \mathfrak{x}_2^j\right) \mathfrak{x}_1 \mathfrak{x}_2^{m-k} + \sum_{k=0}^{m}(-1)^k \begin{pmatrix} m \\ k \end{pmatrix} \mathfrak{x}_2^k \mathfrak{z} \mathfrak{x}_2^{m-k}\right.$$

$$\left. + \sum_{k=0}^{m-1}(-1)^m \begin{pmatrix} m \\ k \end{pmatrix} \mathfrak{x}_2^k \mathfrak{x}_1 \left(\sum_{r+s=m-k-1} \mathfrak{x}_2^r \mathfrak{w} \mathfrak{x}_2^s\right), \sum_{k=0}^{m}(-1)^k \begin{pmatrix} m \\ k \end{pmatrix} \mathfrak{x}_2^k \mathfrak{x}_1 \mathfrak{x}_2^{m-k}\right]$$

for all $\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{w}, \mathfrak{z} \in \mathfrak{S}$ and hence it is satisfied by $\mathcal{Q}$ [2, Theorem 6.4.4]. Thus, $\mathcal{Q}$ is non-commutative as $\mathfrak{S}$. Let us take $\delta \in \mathcal{Q}$ with $\delta \notin \mathfrak{C}$. Also, we can see that $\phi : \mathcal{Q} \to \mathcal{Q}$ is a nonzero derivation of $\mathcal{Q}$ defined by $\phi(\mathfrak{x}_1) = [\delta, \mathfrak{x}_1]$ $\forall$ $\mathfrak{x}_1 \in \mathcal{Q}$. Replacing $\mathfrak{w}, \mathfrak{z}$ by $\phi(\mathfrak{x}_1), \phi(\mathfrak{x}_2)$ in the last expression, we obtain that

$$[\phi([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m]^n = [\phi([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m] \ \forall \ \mathfrak{x}_1, \mathfrak{x}_2 \in \mathcal{Q}.$$

This implies that

$$[[\delta, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m]^n = [[\delta, [\mathfrak{x}_1, \mathfrak{x}_2]_m], [\mathfrak{x}_1, \mathfrak{x}_2]_m].$$

The last relation is identical to (2.1). Using the same method, we arrive at the desired result.

As a result of the preceding theorem, we have the following conclusion.

Corollary 2.1. *Let $\mathfrak{S}$ be a prime ring of characteristic different from two and $\mathfrak{F}$ be a derivation of $\mathfrak{S}$ such that $[\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m]^n = [\mathfrak{F}([\mathfrak{x}_1, \mathfrak{x}_2]_m), [\mathfrak{x}_1, \mathfrak{x}_2]_m]$ $\forall$ $\mathfrak{x}_1, \mathfrak{x}_2 \in \mathfrak{S}$, where m, n are positive integers. Then $\mathfrak{S}$ is commutative.*

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References

[1] A. Alahmadi, S. Ali, A. N. Khan, M .S. Khan, *A characterization of generalized derivations on prime rings*, Comm. Algebra, 44(4-5)(2016), 3201-3210.

[2] K. I. Beidar, W. S. Martindale III, A. V. Mikhalev, Rings with Generalized Identities, *Pure and Applied Mathematics, Marcel Dekker 196, New York,* 1996.

[3] H. E. Bell and M. N. Daif, *On derivations and commutativity in prime rings*, Acta Math. Hung., 66(1995), 337-343.

[4] C. L. Chuang, *GPIs having coefficients in Utumi quotient rings*, Proc. Amer. Math. Soc., 103(1988), 723-728.

[5] C. L. Chuang, *Hypercentral derivations*, J. Algebra, 166(1994), 34-71.

[6] V. D. Filippis, M. A. Raza and N. Rehman, *Commutator with idempotent values on multilinear polynomials in prime rings*, Proc. Indian Acad. Sci (Math. Sci.), 127(1)(2017), 91-98.

[7] A. Giambruno, J. Z. Goncalves and A. Mandel, *Rings with algebraic n-Engel elements*, Comm. Algebra, 22(5)(1994), 1685-1701.

[8] I. N. Herstein, *A condition for the commutativity of the rings*, Canad. J. Math., 9(1957), 583-586.

[9] S. Huang, *Derivation with Engel conditions in prime and semiprime rings*, Czechoslovak Math. J., 61(136)(2011), 1135-1140.

[10] N. Jacobson, *Structure theory for algebraic algebras of bounded degree*, Ann. Math., 46(4)(1945), 695-707.

[11] V. K. Kharchenko, *Differential identities of prime rings*, Algebra Logic, 17(1979), 155-168.

[12] C. Lanski, *An Engel condition with derivation*, Proc. Amer. Math. Soc., 118(1993), 731-734.

[13] T. K. Lee, *Generalized derivations of left faithful rings*, Comm. Algebra, 27(8)(1998), 4057-4073.

[14] T. K. Lee, *Semiprime rings with differential identities*, Bull. Inst. Math. Acad. Sin., 20(1992), 27-38.

[15] W. S. Martindale III, *Prime rings satisfying a generalized polynomial identity*, J. Algebra, 12(1969), 576-584.

[16] M. A. Raza and N. Rehman, *A note on an Engel condition with derivations in rings*, Creat. Math. Inform., 26(1)(2017), 19-27.

[17] L. Rowen, Polynomial identities in ring theory, *Pure and Applied Mathematics, vol 84 Academic Press*, New york, 1980.